Conclusion

The success of the fluidized technique in refrigeration will depend, to a large extent, upon whether the suspended graphite particles can increase the thermal conductivity of the suspension and decrease the fluid film resistance on the walls of the heat exchanger as estimated herein. The estimated thermal conductivity increases rapidly with temperature ($T > 19°R$) and carbon concentration ($C > 0.01$). The estimated c_p decreases slightly as C is increased; it is insensitive to temperature. As a result, the convective heat-transfer coefficient for suspension can increase significantly by increasing the suspension operating temperature or concentration; for example, a tenfold increase in heat-transfer coefficient as compared to that of gaseous helium, can be obtained by adding 10% by weight of carbon dust. Suspensions containing other solid particles are under investigation. No attempt has been made yet to size the heat exchangers or radiators for any specific application.

References

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Gyro Pendulum as a Vertical Reference for a Rotating Platform

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Introduction

It is known that a damped gyro pendulum with fixed point of support† will precess about the vertical and asymptotically reach a state of motion with its symmetry axis and angular momentum vector near vertical, thus acting as a vertical sensor. Motion of a gyro pendulum suspended on rotating platform is investigated here to determine its applicability as a vertical sensor.‡ In general, the platform's axis of rotation will not be along the vertical. A requirement is to determine the misalignment of the platform rotation axis from the vertical by observing the gyro pendulum-platform relative motion.

In a common gyro pendulum, alignment of its symmetry axis to the vertical is caused by viscous resistance to motions of this axis transversing the vertical. Thus, in steady state, such resistance vanishes. With the gyro pendulum suspended at a point on a rotating platform, the analogous resistance is platform-oriented and will not vanish with the symmetry axis aligned to vertical, should this occur. This effect, the Coriolis or gyroscopic forces, and the less interesting centrifugal forces are investigated below. It is shown that the resulting steady-state relative motion will be a coning of the symmetry axis about a line slightly displaced from the vertical at the platform's frequency of rotation. The offset is a result of the centrifugal forces. The motion will lag the motion of vertical with respect to the platform and will be directly proportional to the misalignment of the platform rotation axis from the vertical.

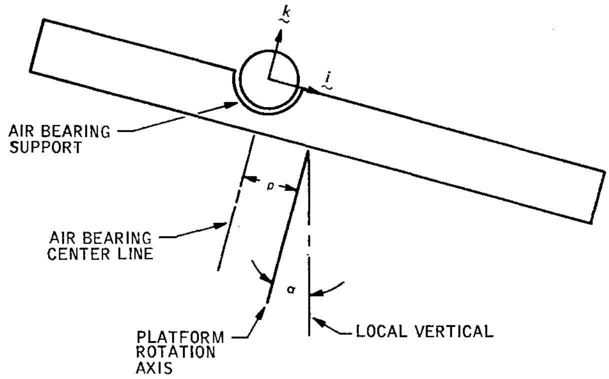

Fig. 1 Geometry at time $t = 0$.

Analysis

Consider a rotating platform with its axis of rotation tilted by a small angle α from the vertical, with an attached air bearing supporting a spherical pendulated rotor at a distance ρ from the axis of rotation (Fig. 1). The platform's centerline is assumed to be parallel to the centerline of the air bearing. A coordinate system **i**, **j**, **k** is fixed to the platform with its origin at the center of symmetry of the air bearing assumed to be the rotor's point of suspension.

The **k** axis is along the platform's axis of rotation. The **i** axis is orthogonal to **k** and in the plane determined by vertical and the platform rotation axis of time $t = 0$. It is assumed that total torque on the rotor about the **k** axis is zero; this is accomplished by spin jets symmetric with respect to this axis. A coordinate system **i′**, **j′**, **k′** is defined so that **k′** is along the rotor symmetry axis and **i′** and **j′** are in the equatorial plane of the rotor, not rotating about **k′** with the rotor (Fig. 2). θ and ϕ are small angles, as indicated. It is assumed that the transverse viscous torques are of the form $-k\dot{\theta}\mathbf{i}' - k\dot{\phi}\,\mathbf{j}'$, where k is an empirical drag coefficient. Such torques are introduced by using a drag pin and viscous dash-pot at the bottom of the rotor. If A is the transverse moment of inertia of the rotor, C is the moment of inertia about the **k′** axis, M is the rotor mass, g is the acceleration of

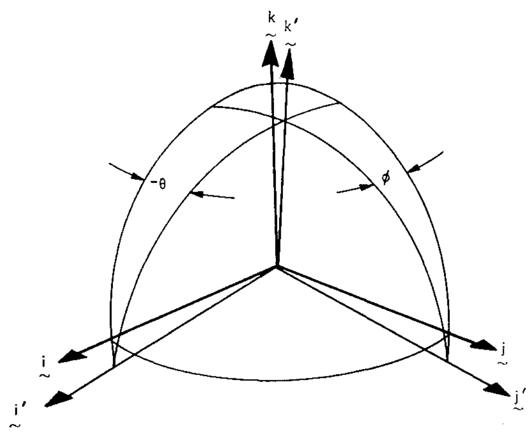

Fig. 2 Coordinate geometry.

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† A gyro pendulum is a symmetrical top with an earth-fixed point of suspension located on its axis of symmetry above the mass center.

‡ In this problem the point of suspension is again fixed in the top on its axis of symmetry above the mass center. However, since the point of suspension is also fixed to the rotating platform, it now rotates with respect to the earth at the platform rotation frequency.

gravity, ω is the platform-rotation rate, h is the gyro angular momentum about the $\mathbf{k}'$ axis, l is the positive distance (in the direction of $\mathbf{k}'$) of the mass center with respect to the support center, and B is $(2A - C)/2$; the equations of motion are, to first order in θ, ϕ, $\dot{\theta}$, and $\dot{\phi}$,

$$A\ddot{\theta} + h\dot{\phi} = Mgl(\theta - \alpha \cos\omega t) + \rho\omega^2 Ml + 2\omega B\dot{\phi} - k\dot{\theta}$$
$$A\ddot{\theta} - h\dot{\theta} = Mgl(\phi + \alpha \sin\omega t) - 2\omega B\theta - k\dot{\phi} \quad (1)$$

Here terms in the centrifugal force of order $Ml\omega^2(\theta^2 + \phi^2)^{1/2}$ have been neglected in comparison to the term $Ml\omega^2\rho$, so that the problem of interest is for ρ much larger than the average excursion of the symmetry axis from the null position. If Θ and Φ designate the Laplace transforms of θ and ϕ, the following equations result in transformation of Eqs. (1):

$$\Theta = \frac{\beta}{\beta^2 + (hs)^2}\left(As\theta_0 + A\dot{\theta}_0 + k\theta_0 + h\phi_0 + \frac{\rho\omega^2 Ml}{s} - Mgl\alpha\frac{s}{s^2+\omega^2}\right) - \frac{hs}{\beta^2 + (hs)^2}\left(As\phi_0 + A\dot{\phi}_0 + k\phi_0 - h\theta_0 + Mgl\alpha\frac{\omega}{s^2+\omega^2}\right)$$
$$\Phi = \frac{hs}{\beta^2 + (hs)^2}\left(As\theta_0 + k\dot{\theta}_0 + h\phi_0 + \frac{\rho\omega^2 Ml}{s} - Mgl\alpha\frac{s}{s^2+\omega^2} + A\dot{\theta}_0\right) + \frac{\beta}{\beta^2 + (hs)2}\left(As\phi_0 + A\dot{\phi}_0 + k\phi_0 - h\theta_0 + Mgl\alpha + \frac{\omega}{s^2+\omega^2}\right) \quad (2)$$

where $\beta = As^2 + ks - Mgl$, the subscript "0" indicates initial values, and s is the Laplace variable. The motion explained by Eqs. (2) covers a large range of phenomena that begins with the two-degree-of-freedom pendulum and ends with the perfect gyroscope. No systematic investigations of these equations are endeavored here, but a certain limiting situation, that of the fast symmetrical top, is investigated.[1]

Motions in the Fast-Top Limit

Motions of the fast-top variety are defined as those for which the quantities $|4MglA|/h^2$ and $|k|/|h|$ are of order ϵ, where ϵ is a small parameter. In what follows, equations to order ϵ for the system characteristic roots are developed. To this end, the denominator $\beta^2 + (hs)^2$ is written as $(\beta + ihs)(\beta - ihs)$, and the quadratic formula is used to write expressions for the four roots. There results estimate roots $Z_j = \zeta_j + i\omega_j$, $j = 1, 2, 3, 4$; good to order ϵ, where Z_1 and Z_3 are complex conjugates, as are Z_2 and Z_4:

$$\zeta_1 = Mglk/h^2 \quad \omega_1 = Mgl/h \qquad \zeta_2 = -k/A \quad \omega_2 = +h/A \quad (3)$$

The first pair of roots Z_1 and Z_3 are associated with precession. Such motions are seen to be stable if Mgl is negative (the mass center should be below the center of support). The second pair of roots Z_2 and Z_4 are associated with nutation and can be shown to have an amplitude that varies inversely as the square of h for a given initial condition. The angular momentum h should be chosen such that the nutation amplitude is sufficiently small for a given initial disturbance. On the other hand, there are advantages to choosing h not excessively large in order that the precession frequency (and its associated damping constant) have a large value, compared to ω.

It is noted that the steady-state centrifugal forces cause a hang off in the θ direction. Further, using Eq. (2), it can be shown that the motion will lag the motion of gravity with respect to the platform and will have an amplitude proportional to α, the platform misalignment. Thus the device can be calibrated to determine this angle α. A slightly involved system using lead networks could be built so that the device would indicate the instantaneous position of vertical with respect to the platform. Such a sensor can be used to determine a vertical reference on a rotating platform which does not maintain its vertical alignment, for example on a portable radar antenna.

In considering the effect of earth's rotation it can be shown that the "effective vertical" is in the direction of the vector $-(Mgl/h)G + \omega_E$, where G is a unit vertical vector and ω_E is a vector in the direction of the earth's rotation axis, with magnitude equal to the rate of earth's rotation.

That is, the angle α measured by the device will be the angle between the foregoing vector and the platform rotation axis. As stated previously, it is desirable to have the basic precession rate Mgl/h large compared with the platform rotation rate. If this is as slow as several rotations per minute, the effect of the earth's rotation will be negligible.

Reference

[1] Goldstein, H., *Classical Mechanics* (Addison-Wesley Press, Inc., Cambridge, Massachusetts, 1950), Chap. V, pp. 164–178.

An Analog Computer for the Skyscreen System of Missile Range Instrumentation

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Two or more precisely oriented phototheodolites (e.g., Wild BC-4) with shutter synchronized in time are often used to obtain missile trajectory data by triangulation methods. The skyscreen system[1–3] solves the problem of tracking a missile and taking multiple exposures in daylight. The Skyscreens are two crossed curtains that cover the photographic plate, each having a narrow slit (as in a focal plane shutter). Thus, only a small area of the plate is exposed where the two slits cross. If the curtains are then made to track the image of the missile, many separate exposures may be made without overexposing the whole plate. This note describes the computer and servo drives that were developed to accept missile bearing and elevation information from a tracking instrument and position the curtains properly.

The problem of converting from relative azimuth and elevation orientation of the tracking instrument to rectangular coordinates on the photographic plate has been analyzed in detail in Ref. 1, from which Eqs. (44A) are simplified for the present purposes through the following assumptions: 1) the azimuth angle of the principal axis (centerline) of the camera is the zero or reference azimuth angle for the computer; and 2) the camera and the tracking device are leveled and are relatively close together, so that their azimuth-elevation coordinate systems are considered to be identical

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